Charge and Current Neutralization of an Ion-Beam Pulse Propagating in a Background Plasma along a Solenoidal Magnetic Field

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The analytical studies show that the application of a small solenoidal magnetic field can drastically change the self-magnetic and self-electric fields of the beam pulse propagating in a background plasma. Theory predicts that when $\omega_{ce} \sim \omega_{pe}\beta_b$, where ω_{ce} is the electron gyrofrequency, ω_{pe} is the electron plasma frequency, and β_b is the ion-beam velocity relative to the speed of light, there is a sizable enhancement of the self-electric and self-magnetic fields due to the dynamo effect. Furthermore, the combined ion-beam–plasma system acts as a paramagnetic medium; i.e., the solenoidal magnetic field inside the beam pulse is enhanced.

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Background plasma can be used as an effective neutralization scheme to transport and compress intense charged particle beam pulses and is used in many applications involving the transport of fast particles in plasmas, including astrophysics [1], accelerator applications [2], and inertial fusion, in particular, fast ignition [3] and heavy ion fusion [4]. The application of a solenoidal magnetic field allows additional control and focusing of the beam pulse. A strong magnetic lens with a magnetic field up to a few Tesla can effectively focus beam pulses in short distances of order a few tens of centimeters. However, the magnetic field can also affect the degree of charge and current neutralization. In this Letter, we show that even a small solenoidal magnetic field, less than 100 G, strongly changes the self-fields in the beam pulse propagating in a background plasma. Such values of magnetic field can be present over distances of a few meters from the strong solenoid, and affect the focusing of the beam pulse. Moreover, a small solenoidal magnetic field can be applied to optimize beam propagation through a background plasma over long distances.

In Refs. [5], the response of a magnetized plasma to intense ion-beam injection was studied while neglecting electron inertia effects, which corresponded to magnetic fields of a few Tesla in ion ring devices. In the present Letter, we analyze the opposite limit, corresponding to small values of magnetic field. In the collisionless limit and without an applied solenoidal magnetic field, the return current is driven by an inductive electric field which is balanced by electron inertia effects [6]. Taking electron inertia effects into account allows us to determine the conditions under which the applied solenoidal magnetic field begins to affect the return current in the plasma, and to reveal the range of magnetic field values that strongly affect the self-electric and self-magnetic fields of a beam pulse propagating in a background plasma.

In a previous study, we developed reduced nonlinear models that describe the stationary plasma disturbance (in the beam frame) excited by the intense ion-beam pulse [6]. The model predicts very good charge neutralization during quasi-steady-state propagation, provided the beam is nonrelativistic and the beam pulse duration is much longer than the electron plasma period, i.e., $\tau_b\omega_{pe} \gg 2\pi$. Here, $\omega_{pe} = (4\pi e^2 n_p/m)^{1/2}$ is the electron plasma frequency, and n_p is the background plasma density.

A high solenoidal magnetic field inhibits radial electron transport, and the electrons move primarily along the magnetic field lines. For high-intensity beam pulses propagating through a background plasma with pulse duration much longer than the electron plasma period, one can assume that the quasineutrality condition holds, $n_e \cong n_p + Z_b n_b$, where n_e is the electron density, n_b is the density of the beam pulse, $Z_b e$ is ion charge for the beam ions, whereas $Z_b = -1$ for electron beams, and n_p is the density of the background plasma ions. For one-dimensional electron motion, the charge density continuity equation, $\partial\rho/\partial t + \nabla\cdot\mathbf{J} = 0$, combined with the quasineutrality condition $[\rho = e(n_p + Z_b n_b - n_e) \cong 0]$ yields $\mathbf{J} \cong \mathbf{0}$; i.e., in this motion, the electrons tend to neutralize the current as well as the charge. Therefore, in the limit of a strong solenoidal magnetic field, the beam current can be expected to be completely neutralized. However, the above description neglects the electron rotation that develops in the presence of a solenoidal magnetic field. Because of the inward radial electron motion, the electrons can enter into the region of smaller magnetic flux. Because of the conservation of canonical angular momentum, the electrons start rotating about the solenoid axis with a very high azimuthal velocity. This rotation produces several unexpected effects.

The first effect is the dynamo effect [7]. If the magnetic field is attached to the electron flow, the electron rotation bends the solenoidal magnetic field lines and generates an azimuthal self-magnetic field, which is much larger than in the limit with no applied solenoidal field. The second effect is the generation of a large radial electric field. Because the

$v_\phi \times B_z$ force should be mostly balanced by a radial electric field, the electron rotation results in a plasma polarization and produces a much larger self-electric field than in the limit with no applied field. The total force acting on the beam particles now can change from always focusing [6] in the limit with no applied solenoidal magnetic field, to defocusing at higher values of solenoidal magnetic field. In particular, an optimum value of magnetic field for long-distance transport of a beam pulse, needed, for example, in inertial fusion applications [4], can be chosen where the forces nearly cancel. The third effect is that the joint system consisting of the ion-beam pulse and the background plasma acts as a paramagnetic medium; i.e., the solenoidal magnetic field is enhanced inside of the ion-beam pulse.

The electron fluid equations together with Maxwell's equations comprise a complete system describing the electron response to the propagating ion-beam pulse. We assume that the beam pulse moves with constant velocity V_b along the z axis. We look for stationary solutions in the reference frame of the moving beam, i.e., where all quantities depend on t and z exclusively through the combination $\zeta = V_b t - z$. Here, for brevity, we consider nonrelativistic beams, although the results can be calculated for relativistic beams as well [8,9]. We further consider cylindrically symmetric, long beam pulses with length l_b and radius r_b, satisfying $l_b \gg V_b/\omega_{pe}$ and $l_b \gg r_b$. The final equations have the simplest form if we express $\mathbf{B} = \nabla \times \mathbf{A}$ and make use of the transverse Coulomb gauge, $\nabla_\perp \cdot \mathbf{A} = \mathbf{0}$. For axisymmetric geometry, this gives $A_r = 0$. The azimuthal magnetic field is $B_\phi = -\partial A_z/\partial r$, and the perturbed (by the plasma) magnetic field components are $B_z = \partial(rA_\phi)/r\partial r$, $B_r = -\partial A_\phi/\partial z$. For long beams with $l_b \gg V_b/\omega_{pe}, r_b$, the displacement current is small compared to the electron current [6], and Ampere's equations are

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = \frac{4\pi e}{c}(Z_b n_b V_{bz} - n_e V_{ez}), \tag{1}$$

$$-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right) = \frac{4\pi e}{c}(Z_b n_b V_{b\phi} - n_e V_{e\phi}). \tag{2}$$

The electron momentum equation can be solved to obtain the components of electron velocity V_{ez}, V_{er}, $V_{e\phi}$. However, it is easier to use conservation of the generalized vorticity [6]

$$\left(\frac{\partial}{\partial t} + \mathbf{V_e}\cdot\nabla\right)\left(\frac{\mathbf{\Omega}}{n_e}\right) = \left(\frac{\mathbf{\Omega}}{n_e}\cdot\nabla\right)\mathbf{V_e}, \tag{3}$$

where the generalized vorticity is defined as

$$\mathbf{\Omega} = \nabla \times (m\mathbf{V_e} - e\mathbf{A}/c). \tag{4}$$

This is a generalization of the "frozen-in" condition for the magnetic field lines, when electron inertia terms are neglected and $\mathbf{\Omega} = -\mathbf{eB}/c$ [10]. For simplicity, we consider the most practically important case when the plasma density is large $n_p \gg n_b$. Because $n_p \gg n_b$, the effects of electron flows are small compared to the beam motion ($V_{ez} \ll V_b$) and can be neglected in Eq. (3) together with variations in the electron density. Substituting $\partial/\partial t = V_b\partial/\partial z$ into Eq. (3), and integrating with zero initial conditions in front of the beam pulse gives $\Omega_\phi = \Omega_z V_{e\phi}/V_b$. Here, we made use of the fact that $\Omega_z = -eB_z/c$ is approximately constant. Note that, if the inertia effects are neglected, this relation describes the magnetic field "frozen in" the electron flow, $B_\phi = B_z V_{e\phi}/V_b$. From Eq. (4), it follows that $\Omega_\phi \simeq -\partial(mV_{ez} - eA_z/c)/\partial r$, where only the radial derivatives are taken into account, due to the approximation of long beam pulses in Eq. (4). Substituting the expressions for Ω_ϕ and Ω_z into $\Omega_\phi = \Omega_z V_{e\phi}/V_b$, and integrating radially gives

$$V_{ez} = \frac{e}{mc}A_z + \frac{eB_z}{mcV_b}\int_r^\infty V_{e\phi}dr. \tag{5}$$

The first term on the right-hand side of Eq. (5) describes the conservation of canonical momentum in the absence of magnetic field; the second term describes the magnetic dynamo effect, i.e., the generation of azimuthal magnetic field due to the rotation of magnetic field lines [7], as shown in Fig. 1. Substituting $\int_r^\infty V_{e\phi}dr$ from Eq. (2), and neglecting very small beam rotation compared to the rotation velocity of the plasma electrons, gives

$$V_{ez} = \frac{e}{mc}A_z - \frac{B_z}{4\pi m V_b n_e}\frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}. \tag{6}$$

Similarly, from the z projection of Eq. (3), we obtain

$$\frac{\partial}{r\partial r}r\left(mV_{e\phi} - \frac{e}{c}A_\phi\right) = -\frac{eB_z}{c}\left(\frac{n_e - n_p}{n_p} - \frac{V_{ez}}{V_b}\right). \tag{7}$$

In order to account for a possible departure from quasineutrality, we substitute into Eq. (7) the perturbation in electron density obtained from

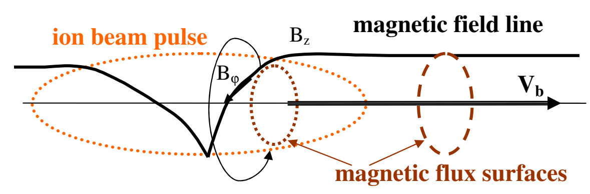

FIG. 1 (color online). Schematic of magnetic field generation due to the dynamo effect. The magnetic field line is shown by the solid (black) line; a contour attached to the electron fluid element is shown by the dashed (brown) line in front of the beam pulse; and the dotted (brown) line indicates this contour inside of the ion-beam pulse, the outline of which is shown by the thin dotted (orange) line. The radial electron displacement generates a poloidal rotation; the poloidal rotation twists the solenoidal magnetic field and generates the poloidal magnetic field.

$$(Z_b n_b - n_e + n_p) = \frac{1}{4\pi e r}\frac{\partial(rE_r)}{\partial r}. \tag{8}$$

In the linear approximation, $n_b \ll n_p$, the radial component of the equation for the electron momenta gives

$$E_r = -\frac{1}{c}V_{e\phi}B_z. \tag{9}$$

Substituting Eqs. (8) and (9) for E_r into Eq. (7), and making use of $J_z = Z_b e n_b V_b - e n_p V_{ez}$ and $\int_r^\infty J_z r dr = (cr/4\pi)\partial A_z/\partial r$, gives after integration of Eq. (7)

$$V_{e\phi}\left(1 + \frac{\omega_{ce}^2}{\omega_{pe}^2}\right) = \frac{e}{mc}A_\phi + \frac{B_z}{4\pi m V_b n_p}\frac{\partial A_z}{\partial r}. \tag{10}$$

Equations (6) and (10), together with Eqs. (1) and (2), compose a full system of equations.

Figure 2 shows a comparison of analytical theory and LSP [11] particle-in-cell (PIC) simulation results for the self-magnetic field, the perturbation in the solenoidal magnetic field, and the radial electric field in the ion-beam pulse. We have performed the PIC simulations in slab geometry, because the numerical noise tends to be larger in cylindrical geometry due to the singularity on the axis ($r = 0$). The beam velocity is $V_b = 0.33c$, and the beam density profile is Gaussian, $n_{b0}\exp(-r^2/r_b^2 - z^2/l_b^2)$, where $r_b = 1$ cm, $l_b = 17$ cm, $n_{b0} = n_p/8 = 3 \times 10^{10}$ cm^{-3}. For this choice of beam parameters, the skin depth is approximately equal to the beam radius $c/\omega_{pe} \simeq r_b$, so that the return current does not screen the beam self-magnetic field significantly. Without the applied solenoidal magnetic field, the maximum value of the self-magnetic field is 14 G, and increases slightly to 15 G for $B_{z0} = 300$ G [see Fig. 2(a)]. However, the maximum value of the self-magnetic field increases strongly to 37 G for $B_{z0} = 900$ G [see Fig. 2(b)]. This is due to the magnetic dynamo effect.

Another unusual effect is that the system consisting of the beam pulse together with the background plasma acts paramagnetically: the solenoidal magnetic field is larger in the center of the beam pulse than the initial value of the applied magnetic field. These effects can be obtained analytically from components of Ampere's equation, Eqs. (1) and (2), in the limit where the skin depth is large compared with the beam radius ($c/\omega_{pe} \gtrsim r_b$). In this limit, the terms proportional to the return current $n_e A_\varphi$ on the right-hand side of Eq. (2) can be neglected compared with the terms on the left-hand side. Without taking into account small ion rotation, and neglecting the term $n_e A_\varphi$, Eq. (2) can then be integrated from $r = 0$ to ∞, assuming that $A_\phi = 0$ as $r \to \infty$ and $\omega_{ce} \gg \omega_{pe}\beta_b$. This gives for the perturbation in the solenoidal magnetic field

$$\delta B_z = \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r} = \frac{4\pi e}{c}\left(\frac{B_z A_z}{4\pi m V_b}\right). \tag{11}$$

Note that δB_z is positive; i.e., the combination of the beam pulse and the plasma acts paramagnetically. Substituting Eq. (11) into Eq. (10) and into Eq. (1) gives

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = \frac{4\pi e J_z}{c}, \tag{12}$$

$$J_z = Z_b n_b V_{bz} - \frac{e n_e}{mc}A_z + \frac{eB_z^2}{4\pi m^2 c V_b^2}A_z. \tag{13}$$

Note that the final positive term on the right-hand side of Eq. (13) proportional to B_z^2 describes the dynamo effect and leads to an increase in the self-magnetic field. This increase becomes significant if $n_e 4\pi m V_b^2/B_z^2 < 1$, or equivalently $\omega_{ce} > \omega_{pe}\beta_b$, where $\omega_{ce} = eB_z/mc$ and $\beta_b = V_b/c$. For the value of the applied magnetic field $B_{z0} = 300G$ in Fig. 2(a), the parameter $\omega_{ce}/\beta_b\omega_{pe} = 0.57$, whereas $\omega_{ce}/\beta_b\omega_{pe} = 1.7$ for $B_{z0} = 900$ G in Fig. 2(b). As a result, the dynamo effect results in a con-

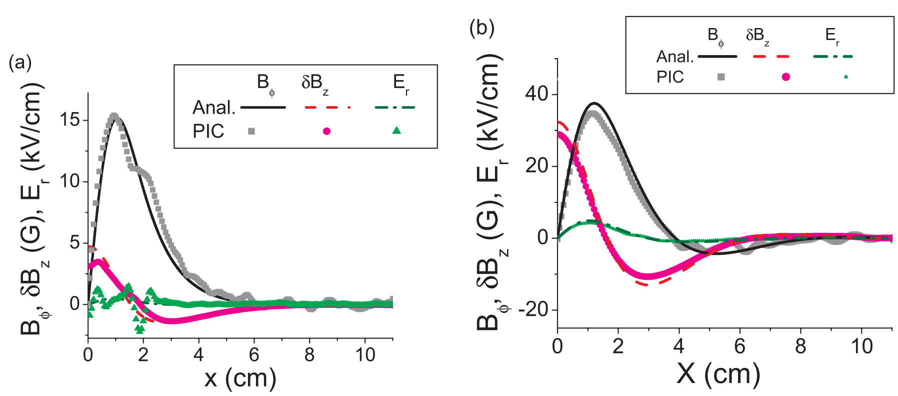

FIG. 2 (color online). Comparison of analytical theory and LSP simulation results for the self-magnetic field, the perturbation in the solenoidal magnetic field, and the radial electric field in a perpendicular slice of the beam pulse. The ion beam moves with velocity $V_b = 0.33c$ along the z axis. The beam density profile is Gaussian with $r_b = 1$ cm, $l_b = 17$ cm, with $n_{b0} = n_p/8 = 3 \times 10^{10}$ cm^{-3}. The values of the applied magnetic field B_{z0} are the following: (a) $B_{z0} = 300$ G; and (b) $B_{z0} = 900$ G.

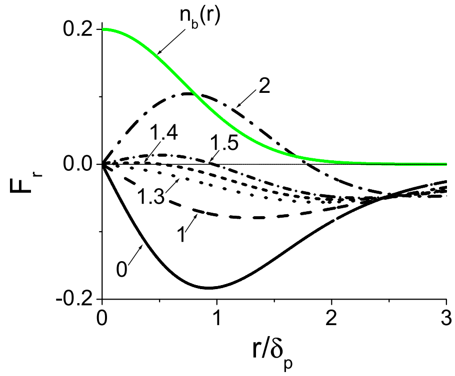

FIG. 3 (color online). The normalized radial force $F_r/(Z_b^2 n_{b0} m V_{bz}^2/n_p \delta_p)$ acting on the beam particles for different values of the parameter $\omega_{ce}^2/\omega_{pe}^2\beta_b^2$. The gray (green) line shows the Gaussian density profile multiplied by 0.2 in order to fit the profile into the plot. The beam radius is equal to the skin depth, $r_b = \delta_p$.

siderable increase in the self-magnetic field of the beam pulse, also in agreement with Eq. (13).

The radial self-electric field is small and cannot be distinguished from numerical noise in the PIC simulations for small values of the applied magnetic field, but increases to the level of a few kV/cm for $B_{z0} = 900$ G. Such a large value of radial electric field can eventually lead to a breakdown of the quasineutrality condition for $\omega_{ce} \gtrsim \omega_{pe}$ and $c/\omega_{pe} \simeq r_b$, which is observed in numerical simulations and can also be derived analytically [8]. In the presence of the solenoidal magnetic field, the radial force acting on the beam ions can change sign from focusing to defocusing, because the radial electric field increases more rapidly than the magnetic force $-Z_b V_{bz} B_\phi$, as the solenoidal magnetic field increases. To demonstrate this tendency analytically, let us consider only linear terms in the radial force equation assuming $n_b \ll n_p$. In this limit, the radial force acting on beam ions, $F_r = eZ_b(-V_{bz}B_\phi/c + E_r)$, becomes

$$F_r = -\frac{eZ_b}{c}(V_{bz}B_\phi + V_{e\phi}B_z), \tag{14}$$

where $V_{e\phi}$ is given by Eq. (10). Figure 3 shows the radial profile of the normalized radial force acting on the beam particles for various values of the parameter $\omega_{ce}^2/\omega_{pe}^2\beta_b^2$. The radial force is nearly zero when $\omega_{ce}^2/\omega_{pe}^2\beta_b^2 = 1.5$ for the main part of the beam pulse. This value can be optimal for beam transport over long distances to avoid the pinching effect. Note that the radial force is focusing at larger radius, which can help to minimize halo particle formation, and produce a tighter beam focus.

In summary, the application of a solenoidal magnetic field strongly affects the degree of current and charge neutralization when $\omega_{ce} > \omega_{pe}\gamma_b\beta_b$, where $\gamma_b = (1-\beta_b^2)^{-1/2}$. This criterion is generalized to the case of relativistic beams in Refs. [8,9]. This threshold value of solenoidal magnetic field is relatively small for nonrelativistic beams. Application of the solenoidal magnetic field leads to three unexpected effects: The first effect is the dynamo effect, in which the electron rotation generates a self-magnetic field that is much larger than in the limit with no applied magnetic field. The second effect is the generation of a large radial electric field. Because the $v_\phi \times B_z$ force should be balanced by a radial electric field, the electron rotation results in a plasma polarization and produces a much larger self-electric field than in the limit with no applied field. The third unexpected effect is that the joint system consisting of the ion-beam pulse and the background plasma acts as a paramagnetic medium; i.e., the solenoidal magnetic field is enhanced inside of the ion-beam pulse. For larger values of the solenoidal magnetic field, corresponding to $\omega_{ce} > 2\gamma_b^2\beta_b\omega_{pe}$, the beam generates whistler and lower-hybrid waves, and this effect is considered in Refs. [8,9].

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[1] H. Alfven, Phys. Rev. **55**, 425 (1939); W. H. Bennett, Phys. Rev. **45**, 890 (1934); M. V. Medvedev and A. Loeb, Astrophys. J. **526**, 697 (1999); A. R. Bell, Mon. Not. R. Astron. Soc. **358**, 181 (2005).

[2] I. Blumenfeld *et al.*, Nature (London) **445**, 741 (2007); P. Chen *et al.*, Phys. Rev. Lett. **54**, 693 (1985); R. Govil *et al.*, Phys. Rev. Lett. **83**, 3202 (1999).

[3] A. J. Kemp *et al.*, Phys. Rev. Lett. **97**, 235001 (2006); R. J. Mason, Phys. Rev. Lett. **96**, 035001 (2006).

[4] P. K. Roy *et al.*, Phys. Rev. Lett. **95**, 234801 (2005); B. G. Logan *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **577**, 1 (2007); I. D. Kaganovich *et al.*, *ibid.* **577**, 93 (2007); A. B. Sefkow *et al.*, *ibid.* **577**, 289 (2007); D. R. Welch, *ibid.* **577**, 231 (2007).

[5] B. V. Oliver, D. D. Ryutov, and R. N. Sudan, Phys. Plasmas **1**, 3383 (1994); B. V. Oliver and R. N. Sudan, Phys. Plasmas **3**, 4725 (1996).

[6] I. D. Kaganovich *et al.*, Phys. Plasmas **8**, 4180 (2001); **11**, 3546 (2004); Phys. Scr. **T107**, 54 (2004); O. Polomarov *et al.*, Phys. Plasmas **14**, 043103 (2007).

[7] R. N. Sudan and P. M. Lyster, Comments Plasma Phys. Control. Fusion **9**, 453 (1984); D. W. Hewett, Nucl. Fusion **24**, 349 (1984).

[8] I. D. Kaganovich, E. A. Startsev, A. B. Sefkow, and R. C. Davidson, Princeton Plasma Physics Laboratory Report No. 4258, 2007.

[9] J. S. Pennington, I. D. Kaganovich, E. A. Startsev, A. B. Sefkow, and R. C. Davidson, in Proceedings of the Particle Accelerator Conference (to be published).

[10] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, New York, 1993).

[11] T. P. Hughes, S. S. Yu, and R. E. Clark, Phys. Rev. ST Accel. Beams **2**, 110401 (1999).